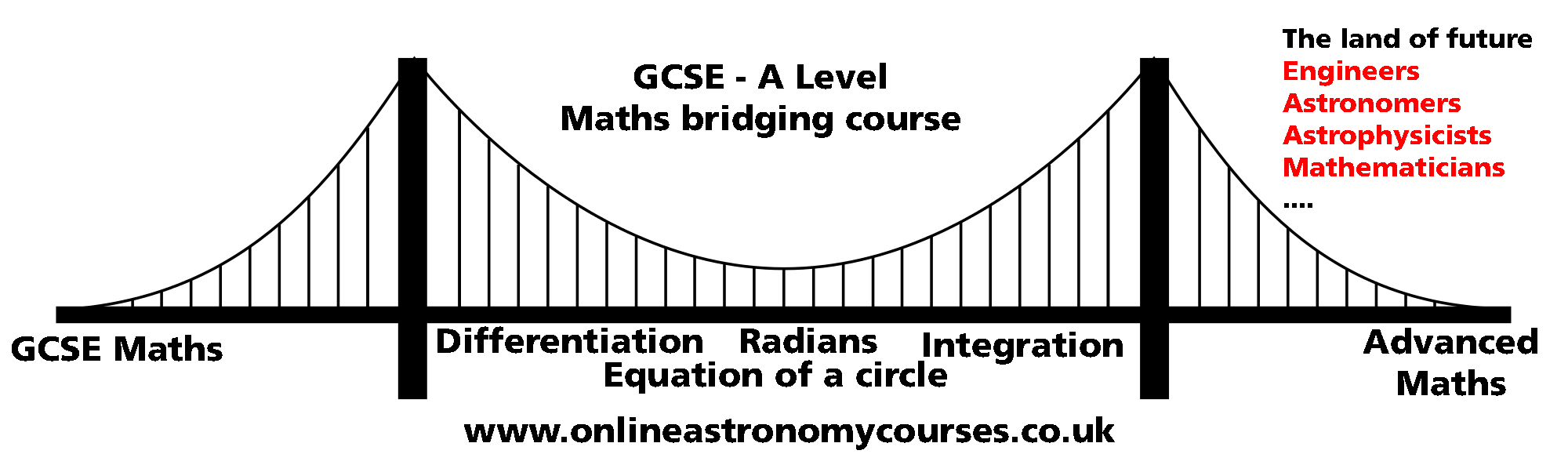


**Introduction to A level maths Induction Booklet 2**

**Part 2 start date 22nd June 2020**

**due in 10th July to Mr Humphreys**



**Solving linear simultaneous equations using the elimination method**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* Two equations are simultaneous when they are both true at the same time.
* Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
* Make sure that the coefficient of one of the unknowns is the same in both equations.
* Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

**Example 1** Solve the simultaneous equations 3*x* + *y* = 5 and *x* + *y* = 1

|  |  |
| --- | --- |
| 3*x* + *y* = 5  *– x* + *y* = 1  2*x* = 4  So *x* = 2  Using *x* + *y* = 1  2 + *y* = 1  So *y* = −1  Check:  equation 1: 3 × 2 + (−1) = 5 YES  equation 2: 2 + (−1) = 1 YES | **1** Subtract the second equation from the first equation to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 2 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve *x* + 2*y* = 13 and 5*x* − 2*y* = 5 simultaneously.

|  |  |
| --- | --- |
| *x* + 2*y* = 13  + 5*x* − 2*y* = 5     6*x* = 18  So *x* = 3  Using *x* + 2*y* = 13  3 + 2*y* = 13  So *y* = 5  Check:  equation 1: 3 + 2 × 5 = 13 YES  equation 2: 5 × 3 − 2 × 5 = 5 YES | **1** Add the two equations together to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 3 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 3** Solve 2*x* + 3*y* = 2 and 5*x* + 4*y* = 12 simultaneously.

|  |  |
| --- | --- |
| (2*x* + 3*y* = 2) × 4  8*x* + 12*y* = 8  (5*x* + 4*y* = 12) × 3 15*x* + 12*y* = 36  7*x* = 28  So *x* = 4  Using 2*x* + 3*y*  = 2  2 × 4 + 3*y* = 2  So *y* = −2  Check:  equation 1: 2 × 4 + 3 × (−2) = 2 YES  equation 2: 5 × 4 + 4 × (−2) = 12 YES | **1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of *y* the same for both equations. Then subtract the first equation from the second equation to eliminate the *y* term.  **2** To find the value of *y*, substitute *x*= 4 into one of the original equations.  **3** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** 4*x* + *y* = 8 **2** 3*x* + *y* = 7

*x* + *y* = 5 3*x* + 2*y* = 5

**3** 4*x* + *y* = 3 **4** 3*x* + 4*y* = 7

3*x* – *y* = 11 *x* – 4*y* = 5

**5** 2*x* + *y* = 11 **6** 2*x* + 3*y* = 11

*x* – 3*y* = 9 3*x* + 2*y* = 4

**Solving linear simultaneous equations using the substitution method**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

**Textbook:**Pure Year 1, 3.1 Linear simultaneous equations

Key points

* The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

**Example 4** Solve the simultaneous equations *y* = 2*x* + 1 and 5*x* + 3*y* = 14

|  |  |
| --- | --- |
| 5*x* + 3(2*x* + 1) = 14  5*x* + 6*x* + 3 = 14  11*x* + 3 = 14  11*x* = 11  So *x* = 1  Using *y* = 2*x* + 1  *y* = 2 × 1 + 1  So *y* = 3  Check:  equation 1: 3 = 2 × 1 + 1 YES  equation 2: 5 × 1 + 3 × 3 = 14 YES | **1** Substitute 2*x* + 1 for *y* into the second equation.  **2** Expand the brackets and simplify.  **3** Work out the value of *x*.  **4** To find the value of *y*, substitute *x*= 1 into one of the original equations.  **5** Substitute the values of *x* and *y* into both equations to check your answers. |

**Example 5** Solve 2*x* − *y* = 16 and 4*x* + 3*y* = −3 simultaneously.

|  |  |
| --- | --- |
| *y* = 2*x* − 16  4*x* + 3(2*x* − 16) = −3  4*x* + 6*x* − 48 = −3  10*x* − 48 = −3  10*x* = 45  So *x* =  Using *y* = 2*x* − 16  *y* = 2 ×  − 16  So *y* = −7  Check:  equation 1: 2 ×  – (–7) = 16 YES  equation 2: 4 ×  + 3 × (−7) = −3 YES | **1** Rearrange the first equation.  **2** Substitute 2*x* − 16 for *y* into the second equation.  **3** Expand the brackets and simplify.  **4** Work out the value of *x*.  **5** To find the value of *y*, substitute *x*=  into one of the original equations.  **6** Substitute the values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**7** *y* = *x* –4 **8** *y* = 2*x* – 3

2*x* + 5*y* = 43 5*x* – 3*y* = 11

**9** 2*y* = 4*x* + 5 **10** 2*x* = *y* – 2

9*x* + 5*y* = 22 8*x* – 5*y* = –11

**11** 3*x* + 4*y* = 8 **12** 3*y* = 4*x* – 7

2*x* – *y* = –13 2*y* = 3*x* – 4

**13** 3*x* = *y* – 1 **14** 3*x* + 2*y* + 1 = 0

2*y* – 2*x* = 3 4*y* = 8 – *x*

Extend

**15** Solve the simultaneous equations 3*x* + 5*y* − 20 = 0 and .

**Solving linear and quadratic simultaneous equations**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* Make one of the unknowns the subject of the linear equation (rearranging where necessary).
* Use the linear equation to substitute into the quadratic equation.
* There are usually two pairs of solutions.

Examples

**Example 1** Solve the simultaneous equations *y* = *x* + 1 and *x*2 + *y*2 = 13

|  |  |
| --- | --- |
| *x*2 + (*x* + 1)2 = 13  *x*2 + *x*2 + *x* + *x* + 1 = 13  2*x*2 + 2*x* + 1 = 13  2*x*2 + 2*x* − 12 = 0  (2*x* − 4)(*x* + 3) = 0  So *x* = 2 or *x* = −3  Using *y* = *x* + 1  When *x* = 2, *y* = 2 + 1 = 3  When *x* = −3, *y* = −3 + 1 = −2  So the solutions are  *x* = 2, *y* = 3 and *x* = −3, *y* = −2  Check:  equation 1: 3 = 2 + 1 YES  and −2 = −3 + 1 YES  equation 2: 22 + 32 = 13 YES  and (−3)2 + (−2)2 = 13 YES | **1** Substitute *x* + 1 for *y* into the second equation.  **2** Expand the brackets and simplify.  **3** Factorise the quadratic equation.  **4** Work out the values of *x*.  **5** To find the value of *y*, substitute both values of *x* into one of the original equations.  **6** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

**Example 2** Solve 2*x* + 3*y* = 5 and 2*y*2 + *xy* = 12 simultaneously.

|  |  |
| --- | --- |
| (*y* + 8)(*y* − 3) = 0  So *y* = −8 or *y* = 3  Using 2*x* + 3*y* = 5  When *y* = −8, 2*x* + 3 × (−8) = 5, *x* = 14.5  When *y* = 3, 2*x* + 3 × 3 = 5, *x* = −2  So the solutions are  *x* = 14.5, *y* = −8 and *x* = −2, *y* = 3  Check:  equation 1: 2 × 14.5 + 3 × (−8) = 5 YES  and 2 × (−2) + 3 × 3 = 5 YES  equation 2: 2×(−8)2 + 14.5×(−8) = 12 YES  and 2 × (3)2 + (−2) × 3 = 12 YES | **1** Rearrange the first equation.  **2** Substitute  for *x* into the second equation. Notice how it is easier to substitute for *x* than for *y*.  **3** Expand the brackets and simplify.  **4** Factorise the quadratic equation.  **5** Work out the values of *y*.  **6** To find the value of *x*, substitute both values of *y* into one of the original equations.  **7** Substitute both pairs of values of *x* and *y* into both equations to check your answers. |

Practice

Solve these simultaneous equations.

**1** *y* = 2*x* + 1 **2** *y* = 6 − *x*

*x*2 + *y*2 = 10 *x*2 + *y*2 = 20

**3** *y* = *x* – 3 **4** *y* = 9 − 2*x*

*x*2 + *y*2 = 5 *x*2 + *y*2 = 17

**5** *y* = 3*x* – 5 **6** *y* = *x* − 5

*y* = *x*2 − 2*x* + 1 *y* = *x*2 − 5*x* – 12

**7** *y* = *x* + 5 **8** *y* = 2*x* – 1

*x*2 + *y*2 = 25 *x*2 + *xy* = 24

**9** *y* = 2*x* **10** 2*x* + *y* = 11

*y*2 – *xy* = 8 *xy* = 15

Extend

**11** *x* – *y* = 1 **12** *y* – *x* = 2

*x*2 + *y*2 = 3 *x*2 + *xy* = 3

**Solving simultaneous equations graphically**

**A LEVEL LINKS**

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples

**Example 1** Solve the simultaneous equations *y* = 5*x* + 2 and *x* + *y* = 5 graphically.

|  |  |
| --- | --- |
| *y* = 5 – *x*  *y* = 5 – *x* has gradient –1 and *y*-intercept 5.  *y* = 5*x* + 2 has gradient 5 and *y*-intercept 2.    Lines intersect at  *x* = 0.5, *y* = 4.5  Check:  First equation *y* = 5*x* + 2:  4.5 = 5 × 0.5 + 2 YES  Second equation *x* + *y* = 5:  0.5 + 4.5 = 5 YES | **1** Rearrange the equation *x* + *y* = 5 to make *y* the subject.  **2** Plot both graphs on the same grid using the gradients and *y*-intercepts.  **3** The solutions of the simultaneous equations are the point of intersection.  **4** Check your solutions by substituting the values into both equations. |

**Example 2** Solve the simultaneous equations *y* = *x* − 4 and *y* = *x*2 − 4*x* + 2 graphically.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | ***x*** | 0 | 1 | 2 | 3 | 4 | | ***y*** | 2 | –1 | –2 | –1 | 2 |     The line and curve intersect at  *x* = 3, *y* = −1 and *x* = 2, *y* = −2  Check:  First equation *y* = *x* − 4:  −1 = 3 − 4 YES  −2 = 2 − 4 YES  Second equation *y* = *x*2 − 4*x* + 2:  −1 = 32 − 4 × 3 + 2 YES  −2 = 22 − 4 × 2 + 2 YES | **1** Construct a table of values and calculate the points for the quadratic equation.  **2** Plot the graph.  **3** Plot the linear graph on the same grid using the gradient and  *y*-intercept. *y* = *x* – 4 has gradient 1 and  *y*-intercept –4.  **4** The solutions of the simultaneous equations are the points of intersection.  **5** Check your solutions by substituting the values into both equations. |

Practice

**1** Solve these pairs of simultaneous equations graphically.

**a** *y* = 3*x* − 1 and *y* = *x* + 3

**b** *y* = *x* − 5 and *y* = 7 − 5*x*

**c** *y* = 3*x* + 4 and *y* = 2 − *x*

**2** Solve these pairs of simultaneous equations graphically.

**a** *x* + *y* = 0 and *y* = 2*x* + 6

**Hint**

Rearrange the equation to make *y* the subject.

**b** 4*x* + 2*y* = 3 and *y* = 3*x* − 1

**c** 2*x* + *y* + 4 = 0 and 2*y* = 3*x* − 1

**3** Solve these pairs of simultaneous equations graphically.

**a** *y* = *x* − 1 and *y* = *x*2 − 4*x* + 3

**b** *y* = 1 − 3*x* and *y* = *x*2 − 3*x* − 3

**c** *y* = 3 − *x* and *y* = *x*2 + 2*x* + 5

**4** Solve the simultaneous equations *x* + *y* = 1 and *x*2 + *y*2 = 25 graphically.

Extend

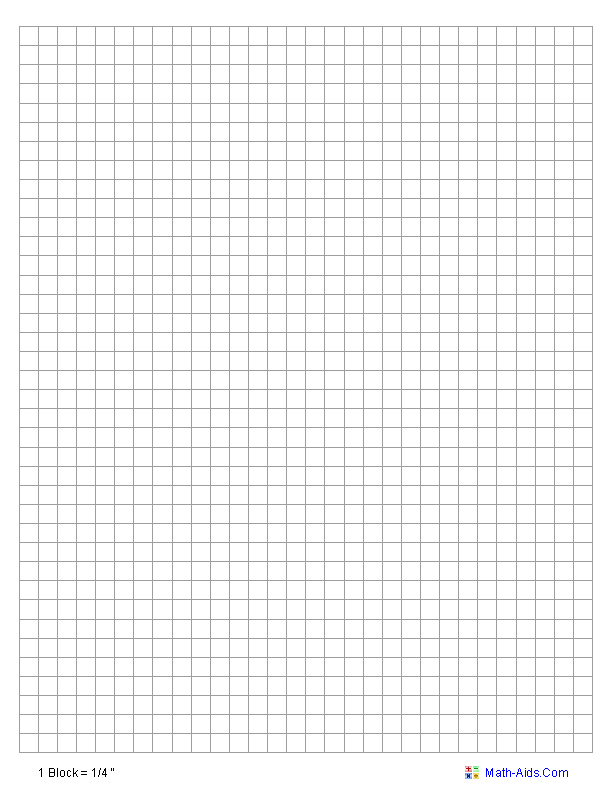
**5** **a** Solve the simultaneous equations 2*x* + *y* = 3 and *x*2 + *y* = 4

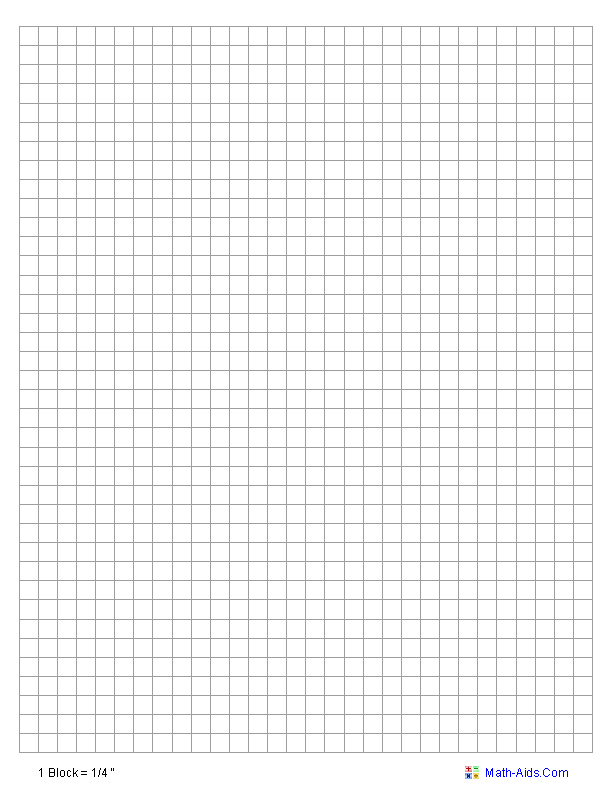
**i** graphically

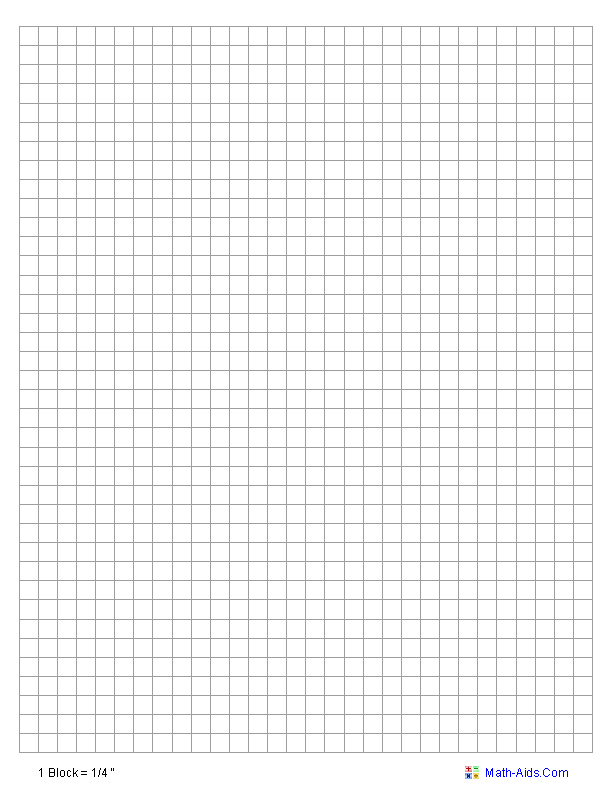
**ii** algebraically to 2 decimal places.

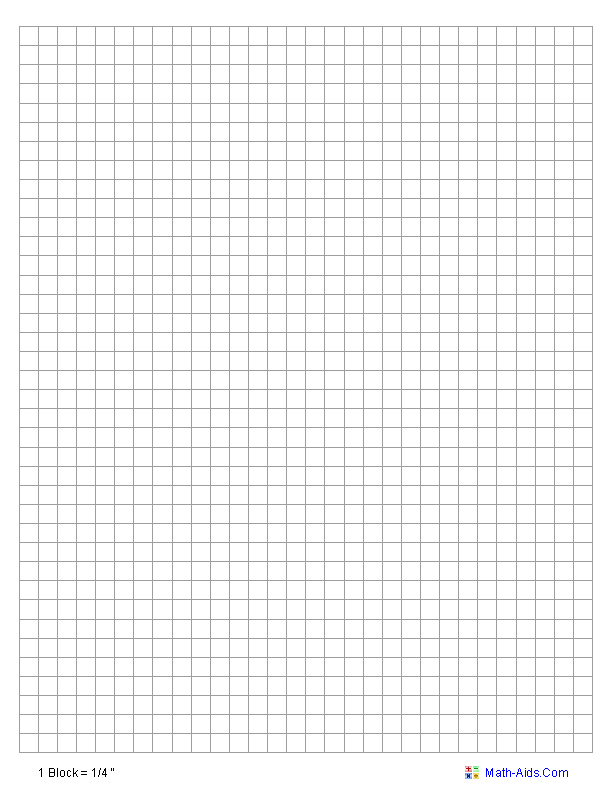
**b** Which method gives the more accurate solutions? Explain your answer.

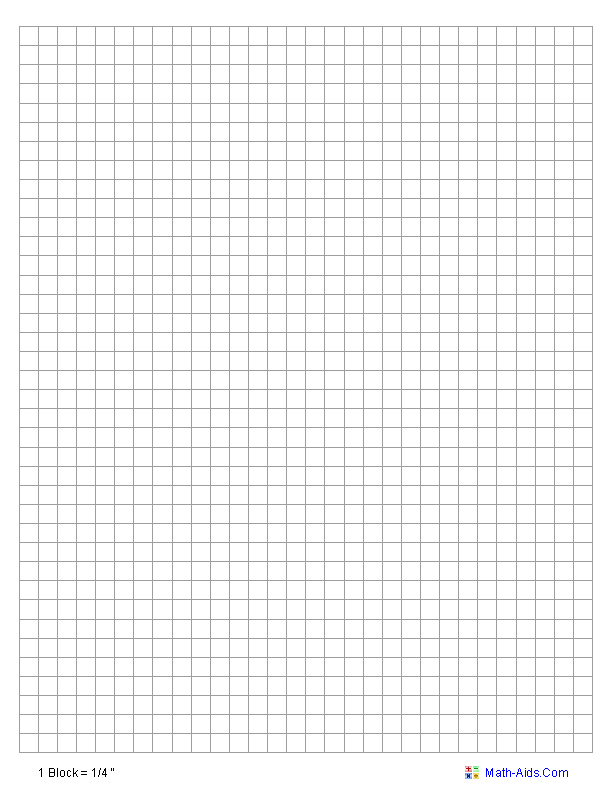
**Room below to sketch each question**

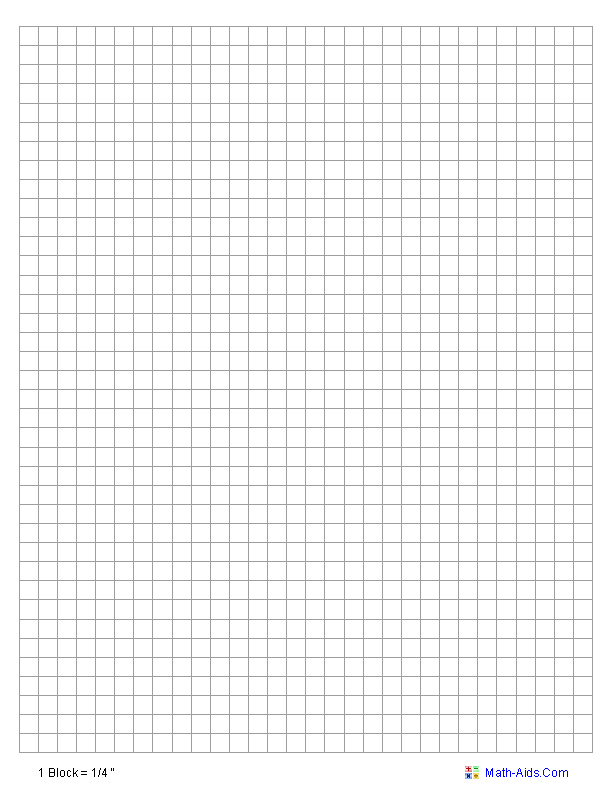












**Linear inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

* Solving linear inequalities uses similar methods to those for solving linear equations.
* When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

**Example 1** Solve −8 ≤ 4*x* < 16

|  |  |
| --- | --- |
| −8 ≤ 4*x* < 16  −2 ≤ *x*  < 4 | Divide all three terms by 4. |

**Example 2** Solve 4 ≤ 5*x* < 10

|  |  |
| --- | --- |
| 4 ≤ 5*x* < 10  ≤ *x* < 2 | Divide all three terms by 5. |

**Example 3** Solve 2*x* − 5 < 7

|  |  |
| --- | --- |
| 2*x* − 5 < 7  2*x* < 12  *x* < 6 | **1** Add 5 to both sides.  **2** Divide both sides by 2. |

**Example 4** Solve 2 − 5*x* ≥ −8

|  |  |
| --- | --- |
| 2 − 5*x* ≥ −8  −5*x* ≥ −10  *x* ≤ 2 | **1** Subtract 2 from both sides.  **2** Divide both sides by −5.  Remember to reverse the inequality when dividing by a negative number. |

**Example 5** Solve 4(*x* − 2) > 3(9 − *x*)

|  |  |
| --- | --- |
| 4(*x* − 2) > 3(9 − *x*)  4*x* − 8 > 27 − 3*x*  7*x* − 8 > 27  7*x* > 35  *x* > 5 | **1** Expand the brackets.  **2** Add 3*x* to both sides.  **3** Add 8 to both sides.  **4** Divide both sides by 7. |

Practice

**1** Solve these inequalities.

**a** 4*x* > 16 **b** 5*x* – 7 ≤ 3 **c** 1 ≥ 3*x* + 4

**d** 5 – 2*x* < 12 **e**  **f** 8 < 3 – 

**2** Solve these inequalities.

**a**  **b** 10 ≥ 2*x* + 3 **c** 7 – 3*x* > –5

**3** Solve

**a** 2 – 4*x* ≥ 18 **b** 3 ≤ 7*x* + 10 < 45 **c** 6 – 2*x* ≥ 4

**d** 4*x* + 17 < 2 – *x* **e**4 – 5*x* < –3*x* **f** –4*x* ≥ 24

**4** Solve these inequalities.

**a** 3*t* + 1 < *t* + 6 **b** 2(3*n* – 1) ≥ *n* + 5

**5** Solve.

**a** 3(2 – *x*) > 2(4 – *x*) + 4 **b** 5(4 – *x*) > 3(5 – *x*) + 2

Extend

**6** Find the set of values of *x* for which 2*x* + 1 > 11 and 4*x* – 2 > 16 – 2*x*.

**Quadratic inequalities**

**A LEVEL LINKS**

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

* First replace the inequality sign by = and solve the quadratic equation.
* Sketch the graph of the quadratic function.
* Use the graph to find the values which satisfy the quadratic inequality.

Examples

**Example 1** Find the set of values of *x* which satisfy *x*2 + 5*x* + 6 > 0

|  |  |
| --- | --- |
| *x*2 + 5*x* + 6 = 0  (*x* + 3)(*x* + 2) = 0  *x* = −3 or *x* = −2    *x* < −3 or *x* > −2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of  *y* = (*x* + 3)(*x* + 2)  **3** Identify on the graph where  *x*2 + 5*x* + 6 > 0, i.e. where *y* > 0  **4** Write down the values which satisfy the inequality *x*2 + 5*x* + 6 > 0 |

**Example 2** Find the set of values of *x* which satisfy *x*2 − 5*x* ≤ 0

|  |  |
| --- | --- |
| *x*2 − 5*x* = 0  *x*(*x* − 5) = 0  *x* = 0 or *x* = 5    0 ≤ *x* ≤ 5 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = *x*(*x* − 5)  **3** Identify on the graph where  *x*2 − 5*x* ≤ 0, i.e. where *y* ≤ 0  **4** Write down the values which satisfy the inequality *x*2 − 5*x* ≤ 0 |

**Example 3** Find the set of values of *x* which satisfy −*x*2 − 3*x* + 10 ≥ 0

|  |  |
| --- | --- |
| −*x*2 − 3*x* + 10 = 0  (−*x* + 2)(*x* + 5) = 0  *x* = 2 or *x* = −5    −5 ≤ *x* ≤ 2 | **1** Solve the quadratic equation by factorising.  **2** Sketch the graph of *y* = (−*x* + 2)(*x* + 5) = 0  **3** Identify on the graph where −*x*2 − 3*x* + 10 ≥ 0, i.e. where *y* ≥ 0  **3** Write down the values which satisfy the inequality −*x*2 − 3*x* + 10 ≥ 0 |

Practice

**1** Find the set of values of *x* for which (*x* + 7)(*x* – 4) ≤ 0

**2** Find the set of values of *x* for which *x*2 – 4*x* – 12 ≥ 0

**3** Find the set of values of *x* for which 2*x*2 –7*x* + 3 < 0

**4** Find the set of values of *x* for which 4*x*2 + 4*x* – 3 > 0

**5** Find the set of values of *x* for which 12 + *x* – *x*2 ≥ 0

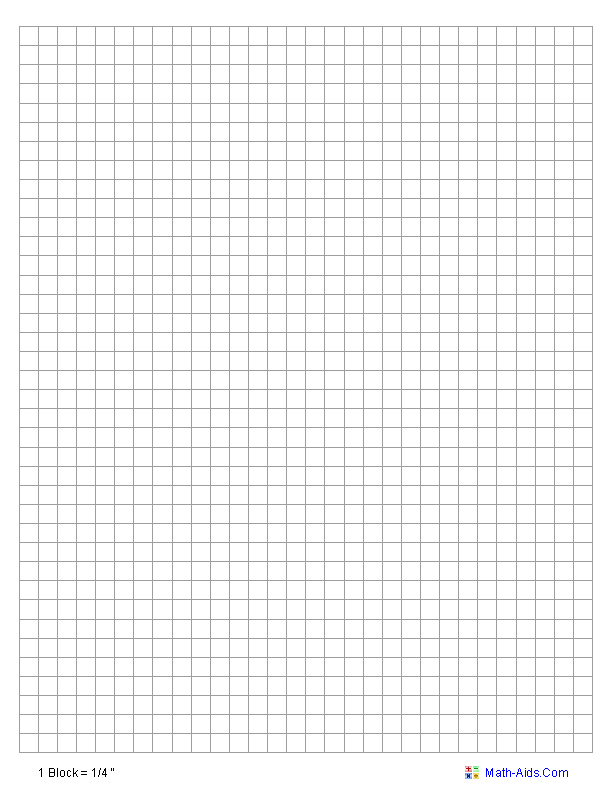
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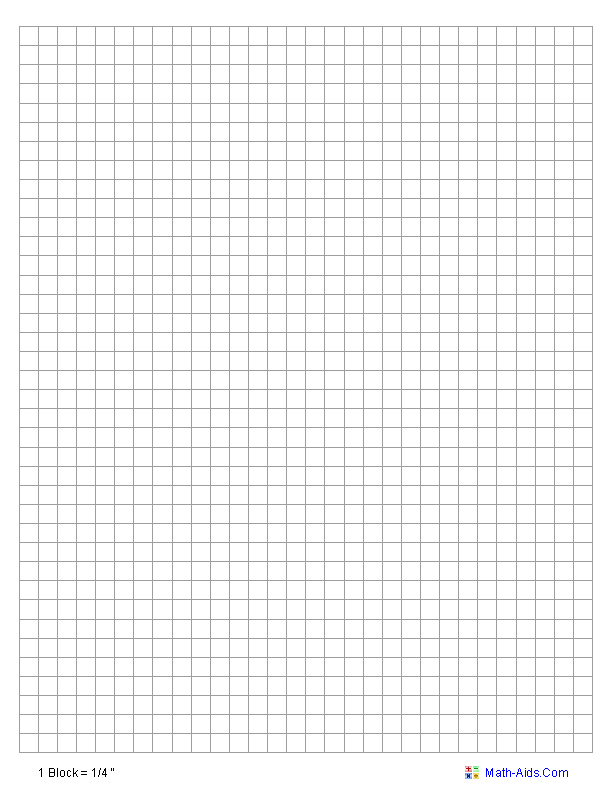
Find the set of values which satisfy the following inequalities.

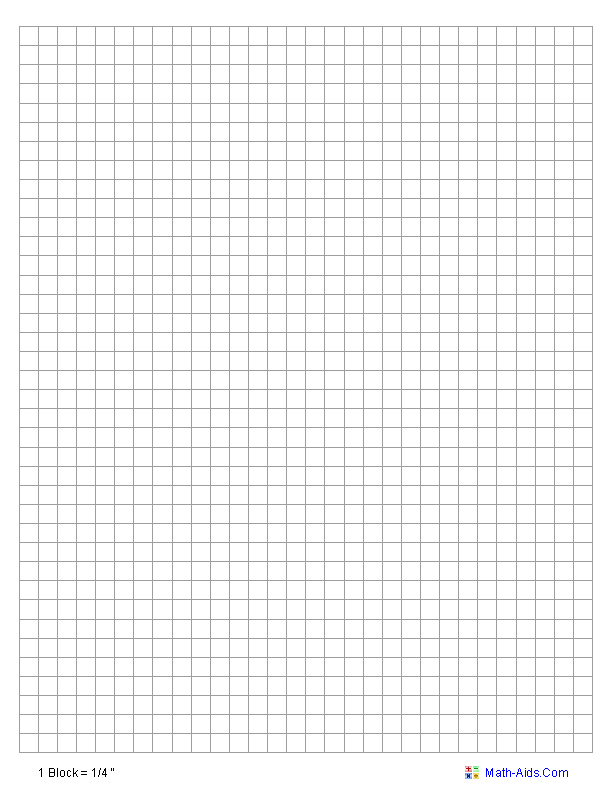
**6** *x*2 + *x* ≤ 6

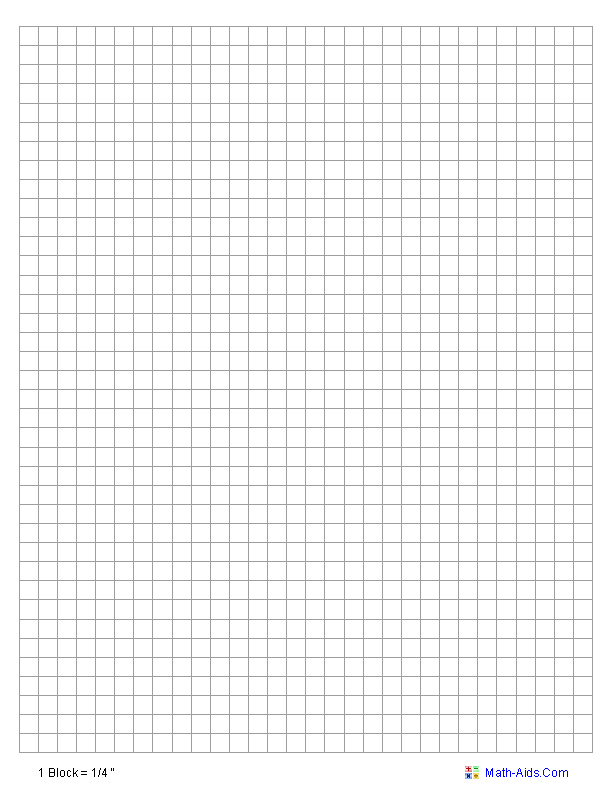
**7** *x*(2*x* – 9) < –10

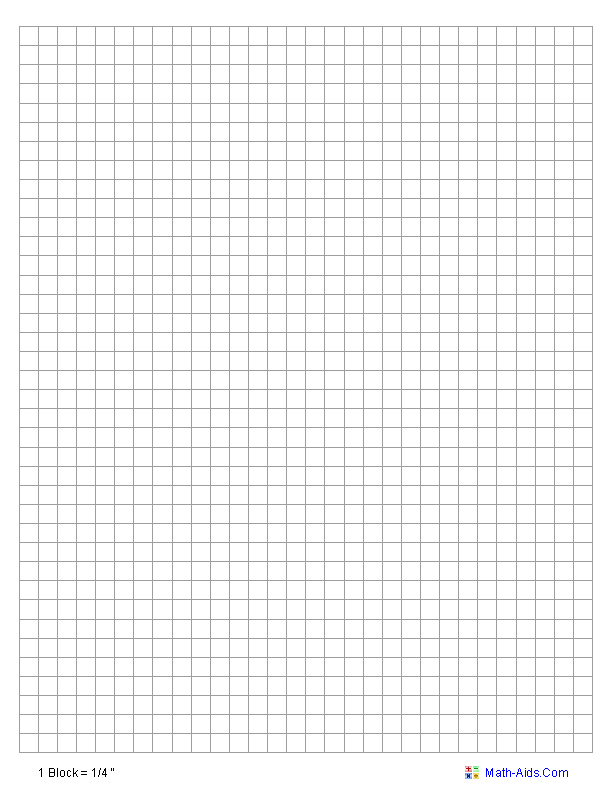
**8** 6*x*2 ≥ 15 + *x*

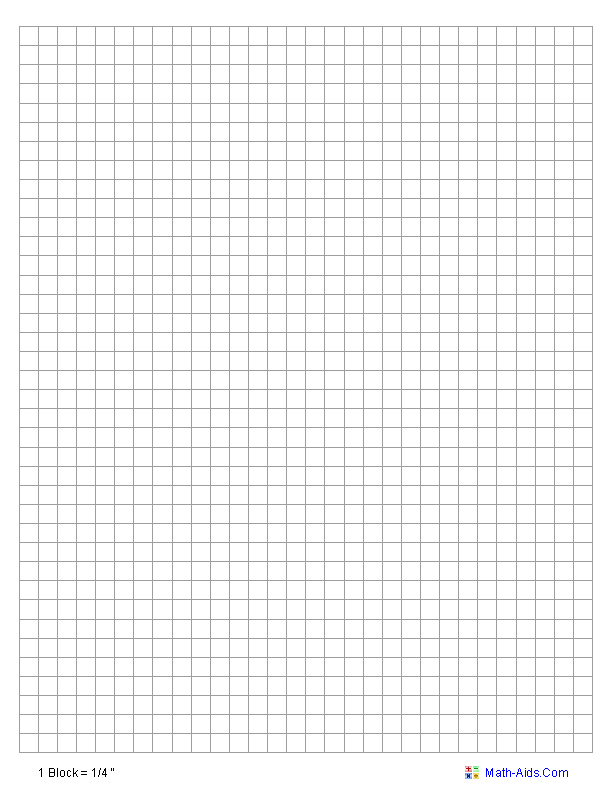


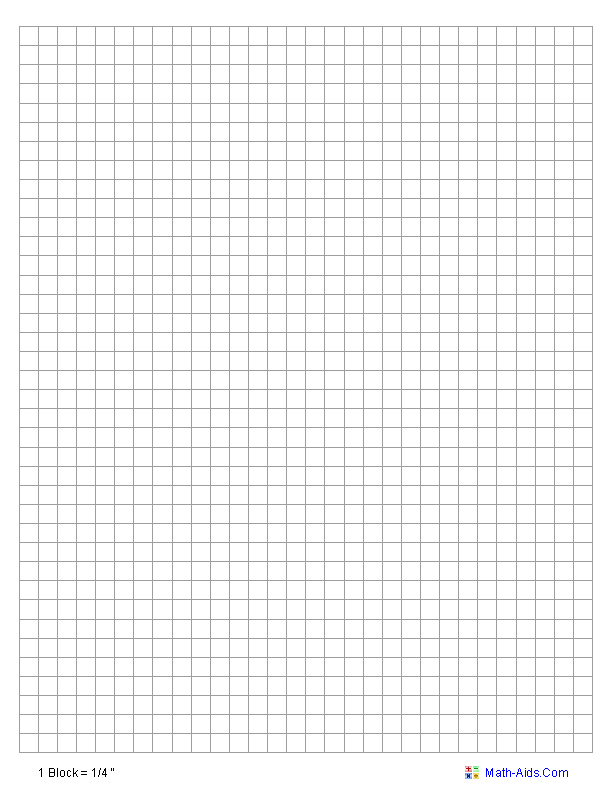


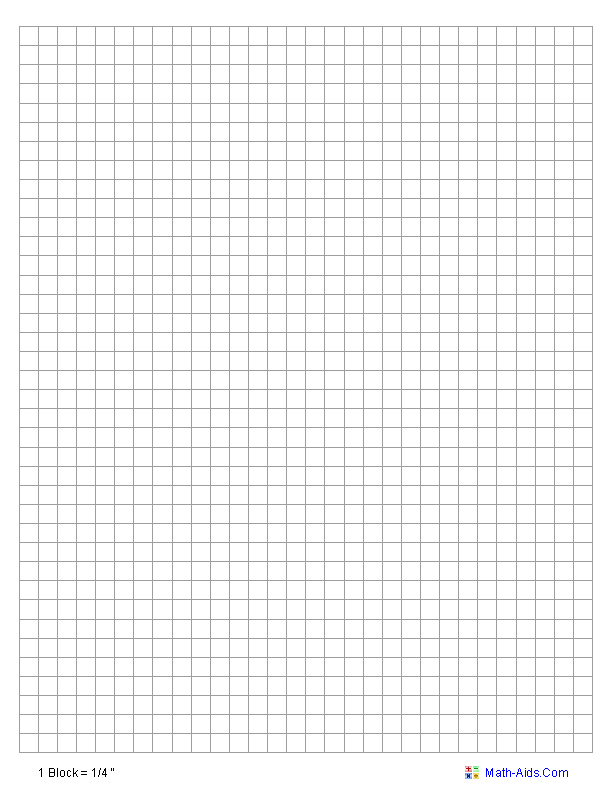


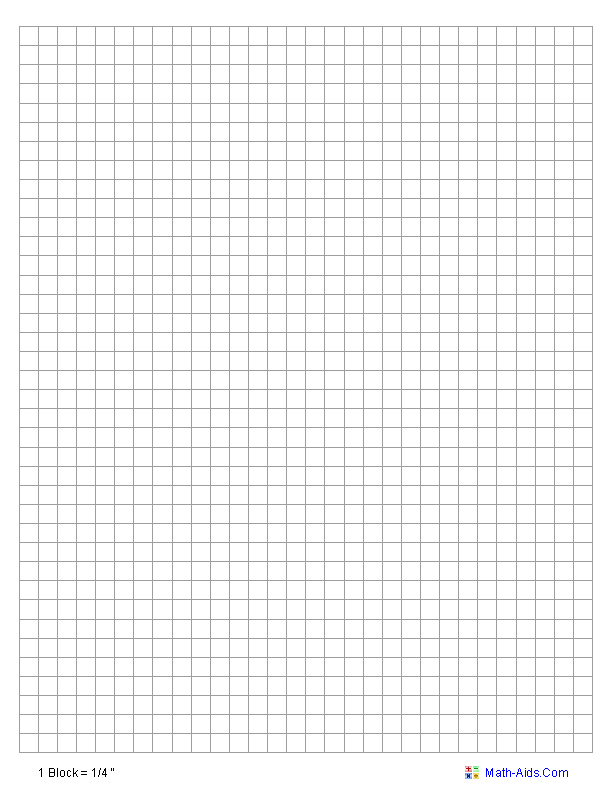








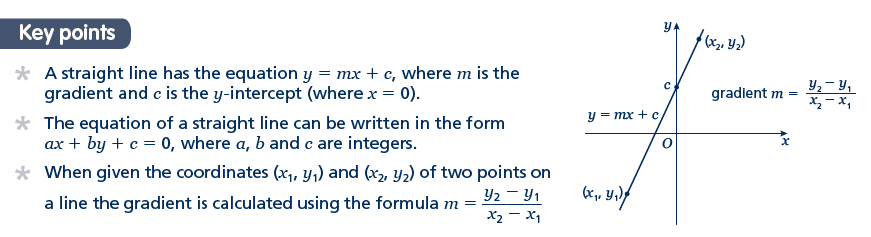




**Straight line graphs**

**A LEVEL LINKS**

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

* A straight line has the equation *y* = *mx* + *c*, where *m* is the gradient and *c* is the *y*-intercept (where *x* = 0).
* The equation of a straight line can be written in the form *ax* + *by* + *c* = 0, where *a*, *b* and *c* are integers.
* When given the coordinates (*x*1, *y*1) and (*x*2, *y*2) of two points on a line the gradient is calculated using the formula 

Examples

**Example 1** A straight line has gradient  and *y*-intercept 3.  
Write the equation of the line in the form *ax* + *by* + *c* = 0.

|  |  |
| --- | --- |
| *m* =  and *c* = 3  So *y* = *x* + 3  *x* + *y* – 3 = 0  *x* + 2*y* − 6 = 0 | **1** A straight line has equation *y*= *mx*+ *c*. Substitute the gradient and *y*-intercept given in the question into thisequation.  **2** Rearrange the equation so all the terms are on one side and 0 is on  the other side.  **3** Multiply both sides by 2 to eliminate the denominator. |

**Example 2** Find the gradient and the *y*-intercept of the line with the equation 3*y* − 2*x* + 4 = 0.

|  |  |
| --- | --- |
| 3*y* − 2*x* + 4 = 0  3*y* = 2*x* − 4    Gradient = *m* =  *y*-intercept = *c* = | **1** Make *y* the subject of the equation.  **2** Divide all the terms by three to get the equation in the form *y* = …  **3** In the form *y* = *mx* + *c*, the gradient is *m* and the *y*-intercept is *c*. |

**Example 3** Find the equation of the line which passes through the point (5, 13) and has gradient 3.

|  |  |
| --- | --- |
| *m* = 3  *y* = 3*x* + *c*  13 = 3 × 5 + *c*  13 = 15 + *c*  *c* = −2  *y* = 3*x* − 2 | **1** Substitute the gradient given in the question into the equation of a straight line *y* = *mx* + *c*.  **2** Substitute the coordinates *x* = 5 and *y* = 13 into the equation.  **3** Simplify and solve the equation.  **4** Substitute *c* = −2 into the equation *y*= 3*x*+ *c* |

**Example 4** Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

|  |  |
| --- | --- |
| , ,  and        *c* = 3 | **1** Substitute the coordinates into the equation  to work out the gradient of the line.  **2** Substitute the gradient into the equation of a straight line *y*= *mx*+ *c*.  **3** Substitute the coordinates of either point into the equation.  **4** Simplify and solve the equation.  **5** Substitute *c* = 3 into the equation |

Practice

**Hint**

Rearrange the equations to the form *y* = *mx* + *c*

**1** Find the gradient and the *y*-intercept of the following equations.

**a** *y* = 3*x* + 5 **b** *y* = *x* – 7

**c** 2*y* = 4*x* – 3 **d** *x* + *y* = 5

**e** 2*x* – 3*y* – 7 = 0 **f** 5*x* + *y* – 4 = 0

**2** Copy and complete the table, giving the equation of the line in the form *y* = *mx* + *c*.

|  |  |  |
| --- | --- | --- |
| **Gradient** | ***y*-intercept** | **Equation of the line** |
| 5 | 0 |  |
| –3 | 2 |  |
| 4 | –7 |  |

**3** Find, in the form *ax* + *by* + *c* = 0 where *a*, *b* and *c* are integers, an equation for each of the lines with the following gradients and *y*-intercepts.

**a** gradient , *y*-intercept –7 **b** gradient 2, *y*-intercept 0

**c** gradient , *y*-intercept 4 **d** gradient –1.2, *y*-intercept –2

**4** Write an equation for the line which passes though the point (2, 5) and has gradient 4.

**5** Write an equation for the line which passes through the point (6, 3) and has gradient 

**6** Write an equation for the line passing through each of the following pairs of points.

**a** (4, 5), (10, 17) **b** (0, 6), (–4, 8)

**c** (–1, –7), (5, 23) **d** (3, 10), (4, 7)

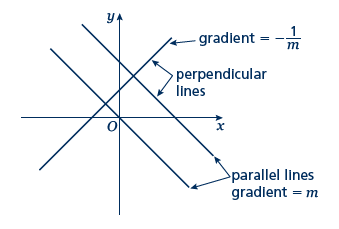
Extend

**7** The equation of a line is 2*y* + 3*x* – 6 = 0.  
Write as much information as possible about this line.

**Parallel and perpendicular lines**

**A LEVEL LINKS**

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

* When lines are parallel they have the same gradient.
* A line perpendicular to the line with equation *y* = *mx* + *c* has gradient .

Examples

**Example 1** Find the equation of the line parallel to *y* = 2*x* + 4 which passes through   
the point (4, 9).

|  |  |
| --- | --- |
| *y* = 2*x* + 4  *m* = 2  *y* = 2*x* + *c*  9 = 2 × 4 + *c*  9 = 8 + *c*  *c* = 1  *y* = 2*x* + 1 | **1** As the lines are parallel they have the same gradient.  **2** Substitute *m* = 2 into the equation of a straight line *y* = *mx* + *c*.  **3** Substitute the coordinates into the equation *y* = 2*x* + *c*  **4** Simplify and solve the equation.  **5** Substitute *c* = 1 into the equation *y*= 2*x* + *c* |

**Example 2** Find the equation of the line perpendicular to *y* = 2*x* − 3 which passes through   
the point (−2, 5).

|  |  |
| --- | --- |
| *y* = 2*x* − 3  *m* = 2        5 = 1 + *c*  *c* = 4 | **1** As the lines are perpendicular, the gradient of the perpendicular line  is .  **2** Substitute *m* =  into *y* = *mx* + *c*.  **3** Substitute the coordinates (–2, 5) into the equation  **4** Simplify and solve the equation.  **5** Substitute *c* = 4 into . |

**Example 3** A line passes through the points (0, 5) and (9, −1).  
Find the equation of the line which is perpendicular to the line and passes through   
its midpoint.

|  |  |
| --- | --- |
| , ,  and        Midpoint = | **1** Substitute the coordinates into the equation  to work out the gradient of the line.  **2** As the lines are perpendicular, the gradient of the perpendicular line  is .  **3** Substitute the gradient into the equation *y* = *mx* + *c*.  **4** Work out the coordinates of the midpoint of the line.  **5** Substitute the coordinates of the midpoint into the equation.  **6** Simplify and solve the equation.  **7** Substitute  into the equation . |

Practice

**1** Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

**a** *y* = 3*x* + 1 (3, 2) **b** *y* = 3 – 2*x* (1, 3)

**c** 2*x* + 4*y* + 3 = 0 (6, –3) **d** 2*y* –3*x* + 2 = 0 (8, 20)

**Hint**

If *m* =  then the negative reciprocal 

**2** Find the equation of the line perpendicular to *y* = *x* – 3 which passes through the point (–5, 3).

**3** Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

**a** *y* = 2*x* – 6 (4, 0) **b** *y* = *x* +  (2, 13)

**c** *x* –4*y* – 4 = 0 (5, 15) **d** 5*y* + 2*x* – 5 = 0 (6, 7)

**4** In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

**a** (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

**5** Work out whether these pairs of lines are parallel, perpendicular or neither.

**a** *y* = 2*x* + 3 **b** *y* = 3*x* **c** *y* = 4*x* – 3  
 *y* = 2*x* – 7 2*x + y* – 3 = 0 4*y* + *x* = 2

**d** 3*x* – *y* + 5 = 0 **e** 2*x* + 5*y* – 1 = 0 **f** 2*x* – *y* = 6

*x* + 3*y* = 1 *y* = 2*x* + 7 6*x* – 3*y* + 3 = 0

**6** The straight line **L1** passes through the points *A* and *B* with coordinates (–4, 4) and (2, 1), respectively.

**a** Find the equation of **L1** in the form *ax* + *by* + *c* = 0

The line **L2** is parallel to the line **L1** and passes through the point *C* with coordinates (–8, 3).

**b** Find the equation of **L2** in the form *ax* + *by* + *c* = 0

The line **L3** is perpendicular to the line **L1** and passes through the origin.

**c** Find an equation of **L3**