

**Introduction to A level maths Induction Booklet 1**

**Part 1 start date 8th June 2020**

**due in 19th June to Mrs Donaldson**



**Expanding brackets
and simplifying expressions**

 **A LEVEL LINKS**

 **Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

**Key points**

* When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
* When you expand two linear expressions, each with two terms of the form *ax* + *b*, where *a*≠ 0 and *b*≠ 0, you create four terms. Two of these can usually be simplified by collecting like terms.

**Examples**

**Example 1** Expand 4(3*x* − 2)

|  |  |
| --- | --- |
| 4(3*x* − 2) = 12*x* − 8 | Multiply everything inside the bracket by the 4 outside the bracket |

**Example 2** Expand and simplify 3(*x* + 5) − 4(2*x* + 3)

|  |  |
| --- | --- |
| 3(*x* + 5) − 4(2*x* + 3) = 3*x* + 15 − 8*x* – 12 = 3 − 5*x* | **1** Expand each set of brackets separately by multiplying (*x* + 5) by 3 and (2*x* + 3) by −4**2** Simplify by collecting like terms: 3*x*− 8*x*= −5*x* and 15 − 12 = 3 |

**Example 3** Expand and simplify (*x* + 3)(*x* + 2)

|  |  |
| --- | --- |
| (*x* + 3)(*x* + 2) = *x*(*x* + 2) + 3(*x* + 2) = *x*2 + 2*x* + 3*x* + 6 = *x*2 + 5*x* + 6 | **1** Expand the brackets by multiplying (*x* + 2) by *x* and (*x* + 2) by 3**2** Simplify by collecting like terms: 2*x*+ 3*x* = 5*x* |

**Example 4** Expand and simplify (*x* − 5)(2*x* + 3)

|  |  |
| --- | --- |
| (*x* − 5)(2*x* + 3) = *x*(2*x* + 3) − 5(2*x* + 3) = 2*x*2 + 3*x* − 10*x* − 15 = 2*x*2 − 7*x* − 15 | **1** Expand the brackets by multiplying (2*x* + 3) by *x* and (2*x* + 3) by −5**2** Simplify by collecting like terms: 3*x*− 10*x* = −7*x* |

**Practice**

**Watch out!**

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is ‘+’; if the signs are different the answer is ‘–’.

**1** Expand.

 **a** 3(2*x* − 1) **b** −2(5*pq* + 4*q*2)

 **c** −(3*xy* − 2*y*2)

**2** Expand and simplify.

 **a** 7(3*x* + 5) + 6(2*x* – 8) **b** 8(5*p* – 2) – 3(4*p* + 9)

 **c** 9(3*s* + 1) –5(6*s* – 10) **d** 2(4*x* – 3) – (3*x* + 5)

**3** Expand.

 **a** 3*x*(4*x* + 8) **b** 4*k*(5*k*2 – 12)

 **c** –2*h*(6*h*2 + 11*h* – 5) **d** –3*s*(4*s*2 – 7*s* + 2)

**4** Expand and simplify.

 **a** 3(*y*2 – 8) – 4(*y*2 – 5) **b** 2*x*(*x* + 5) + 3*x*(*x* – 7)

 **c** 4*p*(2*p* – 1) – 3*p*(5*p* – 2) **d** 3*b*(4*b* – 3) – *b*(6*b* – 9)

**5** Expand (2*y* – 8)

**6** Expand and simplify.

 **a** 13 – 2(*m* + 7) **b** 5*p*(*p*2 + 6*p*) – 9*p*(2*p* – 3)

**7** The diagram shows a rectangle.

 Write down an expression, in terms of *x*, for the

area of the rectangle.

 Show that the area of the rectangle can be written as 21*x*2– 35*x*

**8** Expand and simplify.

 **a** (*x* + 4)(*x* + 5) **b** (*x* + 7)(*x* + 3)

 **c** (*x* + 7)(*x* – 2) **d** (*x* + 5)(*x* – 5)

 **e** (2*x* + 3)(*x* – 1) **f** (3*x* – 2)(2*x* + 1)

 **g** (5*x* – 3)(2*x* – 5) **h** (3*x* – 2)(7 + 4*x*)

 **i** (3*x* + 4*y*)(5*y* + 6*x*) **j** (*x* + 5)2

 **k** (2*x* − 7)2 **l** (4*x* − 3*y*)2

**Extend**

**9** Expand and simplify (*x* + 3)² + (*x* − 4)²

**10** Expand and simplify.

**a** 

**b** 

**Surds and rationalising the denominator**

**A LEVEL LINKS**

 **Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* A surd is the square root of a number that is not a square number,
for example  etc.
* Surds can be used to give the exact value for an answer.
* 
* 
* To rationalise the denominator means to remove the surd from the denominator of a fraction.
* To rationalise you multiply the numerator and denominator by the surd 
* To rationalise  you multiply the numerator and denominator by 

Examples

**Example 1** Simplify 

|  |  |
| --- | --- |
|   | **1** Choose two numbers that are factors of 50. One of the factors must be a square number**2** Use the rule **3** Use  |

**Example 2** Simplify 

|  |  |
| --- | --- |
|  | **1** Simplify  and . Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number**2** Use the rule **3** Use  and **4** Collect like terms |

**Example 3** Simplify 

|  |  |
| --- | --- |
| = = 7 – 2= 5 | **1** Expand the brackets. A common mistake here is to write **2** Collect like terms:  |

**Example 4** Rationalise 

|  |  |
| --- | --- |
|  =  = =  | **1** Multiply the numerator and denominator by **2** Use  |

**Example 5** Rationalise and simplify 

|  |  |
| --- | --- |
|  =  =  =  =  | **1** Multiply the numerator and denominator by **2** Simplify  in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number**3** Use the rule **4** Use **5** Simplify the fraction: simplifies to  |

**Example 6** Rationalise and simplify 

|  |  |
| --- | --- |
|  = = = = =  | **1** Multiply the numerator and denominator by **2** Expand the brackets**3** Simplify the fraction**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1 |

Practice

**Hint**

One of the two numbers you choose at the start must be a square number.

**1** Simplify.

 **a**  **b** 

 **c**  **d** 

 **e**  **f** 

 **g**  **h** 

**2** Simplify.

**Watch out!**

Check you have chosen the highest square number at the start.

 **a**  **b** 

 **c**  **d** 

 **e  f** 

**3** Expand and simplify.

 **a**  **b** 

 **c**  **d** 

**4** Rationalise and simplify, if possible.

 **a**  **b** 

 **c**  **d** 

 **e**  **f** 

 **g**  **h** 

**5** Rationalise and simplify.

 **a**  **b**  **c** 

# **Extend**

**6** Expand and simplify 

**7** Rationalise and simplify, if possible.

 **a**  **b** 

**Rules of indices**

 **A LEVEL LINKS**

 **Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* *am* × *an* = *am* + *n*
* 
* (*am*)*n* = *amn*
* *a*0 = 1
*  i.e. the *n*th root of *a*
* 
* 
* The square root of a number produces two solutions, e.g. .

Examples

**Example 1** Evaluate 100

|  |  |
| --- | --- |
| 100 = 1 | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate 

|  |  |
| --- | --- |
|  = 3 | Use the rule  |

**Example 3** Evaluate 

|  |  |
| --- | --- |
|  =  = 9 | **1** Use the rule **2** Use  |

**Example 4** Evaluate 

|  |  |
| --- | --- |
|   | **1** Use the rule **2** Use  |

**Example 5** Simplify 

|  |  |
| --- | --- |
|  = 3*x*3 | 6 ÷ 2 = 3 and use the rule  to give  |

**Example 6** Simplify 

|  |  |
| --- | --- |
|  = *x*8 − 4 = *x*4 | **1** Use the rule **2** Use the rule  |

**Example 7** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | Use the rule , note that the fraction  remains unchanged |

**Example 8** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | **1** Use the rule **2** Use the rule  |

Practice

**1** Evaluate.

 **a** 140 **b** 30 **c** 50 **d** *x*0

**2** Evaluate.

 **a**  **b**  **c**  **d** 

**3** Evaluate.

 **a**  **b**  **c**  **d** 

**4** Evaluate.

 **a** 5–2 **b** 4–3 **c** 2–5 **d** 6–2

**5** Simplify.

 **a**  **b** 

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule *a*0 = 1.

 **c**  **d** 

 **e**  **f** 

 **g**  **h** 

**6** Evaluate.

 **a**  **b**  **c** 

 **d**  **e**  **f** 

**7** Write the following as a single power of *x*.

 **a**  **b**  **c** 

 **d**  **e**  **f** 

**8** Write the following without negative or fractional powers.

 **a**  **b** *x*0 **c** 

 **d**  **e**  **f** 

**9** Write the following in the form *axn*.

 **a**  **b**  **c** 

 **d**  **e**  **f** 3

# **Extend**

**10** Write as sums of powers of *x*.

 **a**  **b**  **c** 

**Factorising expressions**

 **A LEVEL LINKS**

 **Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Factorising an expression is the opposite of expanding the brackets.
* A quadratic expression is in the form *ax*2 + *bx* + *c*, where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose product is *ac*.
* An expression in the form *x*2 – *y*2 is called the difference of two squares. It factorises to (*x* – *y*)(*x* + *y*).

Examples

**Example 1** Factorise 15*x*2*y*3 + 9*x*4*y*

|  |  |
| --- | --- |
| 15*x*2*y*3 + 9*x*4*y* = 3*x*2*y*(5*y*2 + 3*x*2) | The highest common factor is 3*x*2*y*.So take 3*x*2*y* outside the brackets and then divide each term by 3*x*2*y* to find the terms in the brackets |

**Example 2** Factorise 4*x*2 – 25*y*2

|  |  |
| --- | --- |
| 4*x*2 – 25*y*2 = (2*x* + 5*y*)(2*x* − 5*y*) | This is the difference of two squares as the two terms can be written as (2*x*)2and (5*y*)2 |

**Example 3** Factorise *x*2 + 3*x* – 10

|  |  |
| --- | --- |
| *b* = 3, *ac* = −10So *x*2 + 3*x* – 10 = *x*2 + 5*x* – 2*x* – 10 = *x*(*x* + 5) – 2(*x* + 5) = (*x* + 5)(*x* – 2) | **1** Work out the two factors of *ac*= −10 which add to give *b*= 3 (5 and −2)**2** Rewrite the *b* term (3*x*) using these two factors**3** Factorise the first two terms and the last two terms**4** (*x* + 5) is a factor of both terms |

**Example 4** Factorise 6*x*2 − 11*x* − 10

|  |  |
| --- | --- |
| *b* = −11, *ac* = −60So 6*x*2 − 11*x* – 10 =6*x*2 − 15*x* + 4*x* – 10 = 3*x*(2*x* − 5) + 2(2*x* − 5) = (2*x* – 5)(3*x* + 2) | **1** Work out the two factors of *ac*= −60 which add to give *b*= −11(−15 and 4)**2** Rewrite the *b* term (−11*x*) using these two factors**3** Factorise the first two terms and the last two terms**4** (2*x* − 5) is a factor of both terms |

**Example 5** Simplify 

|  |  |
| --- | --- |
| For the numerator:*b* = −4, *ac* = −21So*x*2 − 4*x* – 21 = *x*2 − 7*x* + 3*x* – 21 = *x*(*x* − 7) + 3(*x* − 7) = (*x* – 7)(*x* + 3)For the denominator:*b* = 9, *ac* = 18So 2*x*2 + 9*x* + 9 = 2*x*2 + 6*x* + 3*x* + 9 = 2*x*(*x* + 3) + 3(*x* + 3) = (*x* + 3)(2*x* + 3)So  =  | **1** Factorise the numerator and the denominator**2** Work out the two factors of *ac*= −21 which add to give *b*= −4(−7 and 3)**3** Rewrite the *b* term (−4*x*) using these two factors**4** Factorise the first two terms and the last two terms**5** (*x* − 7) is a factor of both terms**6** Work out the two factors of *ac*= 18 which add to give *b*= 9 (6 and 3)**7** Rewrite the *b* term (9*x*) using these two factors**8** Factorise the first two terms and the last two terms**9** (*x* + 3) is a factor of both terms**10** (*x* + 3) is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |

Practice

**1** Factorise.

**Hint**

Take the highest common factor outside the bracket.

 **a** 6*x*4*y*3 – 10*x*3*y*4 **b** 21*a*3*b*5 + 35*a*5*b*2

 **c** 25*x*2*y*2 – 10*x*3*y*2 + 15*x*2*y*3

**2** Factorise

 **a** *x*2 + 7*x* + 12 **b** *x*2 + 5*x* – 14

 **c** *x*2 – 11*x* + 30 **d** *x*2 – 5*x* – 24

 **e** *x*2 – 7*x* – 18 **f** *x*2 + *x* –20

 **g** *x*2 – 3*x* – 40 **h** *x*2 + 3*x* – 28

**3** Factorise

 **a** 36*x*2 – 49*y*2 **b** 4*x*2 – 81*y*2

 **c** 18*a*2 – 200*b*2*c*2

**4** Factorise

 **a** 2*x*2 + *x* –3 **b** 6*x*2 + 17*x* + 5

 **c** 2*x*2 + 7*x* + 3 **d** 9*x*2 – 15*x* + 4

 **e** 10*x*2 + 21*x* + 9 **f** 12*x*2 – 38*x* + 20

**5** Simplify the algebraic fractions.

 **a**  **b** 

 **c**  **d** 

 **e**  **f** 

**6** Simplify

 **a**  **b** 

 **c**  **d** 

# **Extend**

**7** Simplify 

**8** Simplify **Completing the square**

 **A LEVEL LINKS**

 **Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Completing the square for a quadratic rearranges *ax*2 + *bx* + *c* into the form *p*(*x* + *q*)2 + *r*
* If *a* ≠ 1, then factorise using *a* as a common factor.

Examples

**Example 1** Complete the square for the quadratic expression *x*2 + 6*x* − 2

|  |  |
| --- | --- |
| *x*2 + 6*x* − 2= (*x* + 3)2 − 9 − 2= (*x* + 3)2 − 11 | **1** Write *x*2 + *bx* + *c* in the form **2** Simplify |

**Example 2** Write 2*x*2 − 5*x* + 1 in the form *p*(*x* + *q*)2 + *r*

|  |  |
| --- | --- |
| 2*x*2 − 5*x* + 1= = = =  | **1** Before completing the square write *ax*2 + *bx* + *c* in the form **2** Now complete the square by writing  in the form **3** Expand the square brackets – don’t forget to multiply by the factor of 2**4** Simplify |

Practice

1 Write the following quadratic expressions in the form (*x* + *p*)2 + *q*

 **a** *x*2 + 4*x* + 3 **b** *x*2 – 10*x* – 3

 **c** *x*2 – 8*x* **d** *x*2 + 6*x*

 **e** *x*2 – 2*x* + 7 **f** *x*2 + 3*x* – 2

**2** Write the following quadratic expressions in the form *p*(*x* + *q*)2 + *r*

 **a** 2*x*2 – 8*x* – 16 **b** 4*x*2 – 8*x* – 16

 **c** 3*x*2 + 12*x* – 9 **d** 2*x*2 + 6*x* – 8

**3** Complete the square.

 **a** 2*x*2 + 3*x* + 6 **b** 3*x*2 – 2*x*

 **c** 5*x*2 + 3*x* **d** 3*x*2 + 5*x* + 3

Extend

**4** Write (25*x*2 + 30*x* + 12) in the form (*ax* + *b*)2 + *c*.

**Solving quadratic equations by factorisation**

 **A LEVEL LINKS**

 **Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* A quadratic equation is an equation in the form *ax*2 + *bx* + *c* = 0 where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose products is *ac*.
* When the product of two numbers is 0, then at least one of the numbers must be 0.
* If a quadratic can be solved it will have two solutions (these may be equal).

Examples

**Example 1** Solve 5*x*2 = 15*x*

|  |  |
| --- | --- |
| 5*x*2 = 15*x*5*x*2 − 15*x* = 05*x*(*x* − 3) = 0So 5*x* = 0 or (*x* − 3) = 0Therefore *x* = 0 or *x* = 3 | **1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by *x* as this would lose the solution *x* = 0.**2** Factorise the quadratic equation. 5*x* is a common factor.**3** When two values multiply to make zero, at least one of the values must be zero.**4** Solve these two equations. |

**Example 2** Solve *x*2 + 7*x* + 12 = 0

|  |  |
| --- | --- |
| *x*2 + 7*x* + 12 = 0*b* = 7, *ac* = 12*x*2 + 4*x* + 3*x* + 12 = 0*x*(*x* + 4) + 3(*x* + 4) = 0(*x* + 4)(*x* + 3) = 0So (*x* + 4)= 0 or (*x* + 3) = 0Therefore *x* = −4 or *x* = −3 | **1** Factorise the quadratic equation. Work out the two factors of *ac* = 12 which add to give you *b* = 7. (4 and 3)**2** Rewrite the *b* term (7*x*) using these two factors.**3** Factorise the first two terms and the last two terms.**4** (*x* + 4) is a factor of both terms.**5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations. |

**Example 3** Solve 9*x*2 − 16 = 0

|  |  |
| --- | --- |
| 9*x*2 − 16 = 0(3*x* + 4)(3*x* – 4) = 0So (3*x* + 4) = 0 or (3*x* – 4) = 0 or  | **1** Factorise the quadratic equation. This is the difference of two squares as the two terms are (3*x*)2 and (4)2.**2** When two values multiply to make zero, at least one of the values must be zero.**3** Solve these two equations. |

**Example 4** Solve 2*x*2 − 5*x* − 12 = 0

|  |  |
| --- | --- |
| *b* = −5, *ac* = −24So 2*x*2 − 8*x* + 3*x* – 12 = 02*x*(*x* − 4) + 3(*x* − 4) = 0(*x* – 4)(2*x* + 3) = 0So (*x* – 4) = 0 or (2*x* +3) = 0 or  | **1** Factorise the quadratic equation.Work out the two factors of *ac* = −24 which add to give you *b* = −5. (−8 and 3)**2** Rewrite the *b* term (−5*x*) using these two factors.**3** Factorise the first two terms and the last two terms.**4** (*x* − 4) is a factor of both terms.**5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations. |

Practice

**1** Solve

 **a** 6*x*2 + 4*x* = 0 **b** 28*x*2 – 21*x* = 0

**Hint**

Get all terms onto one side of the equation.

 **c** *x*2 + 7*x* + 10 = 0 **d** *x*2 – 5*x* + 6 = 0

 **e** *x*2 – 3*x* – 4 = 0 **f** *x*2 + 3*x* – 10 = 0

 **g** *x*2 – 10*x* + 24 = 0 **h** *x*2 – 36 = 0

 **i** *x*2 + 3*x* – 28 = 0 **j** *x*2 – 6*x* + 9 = 0

 **k** 2*x*2 – 7*x* – 4 = 0 **l** 3*x*2 – 13*x* – 10 = 0

**2** Solve

 **a** *x*2 – 3*x* = 10 **b** *x*2 – 3 = 2*x*

 **c** *x*2 + 5*x* = 24 **d** *x*2 – 42 = *x*

 **e** *x*(*x* + 2) = 2*x* + 25 **f** *x*2 – 30 = 3*x* – 2

 **g** *x*(3*x* + 1) = *x*2 + 15 **h** 3*x*(*x* – 1) = 2(*x* + 1)

**Sketching quadratic graphs**

 **A LEVEL LINKS**

 **Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* The graph of the quadratic function
*y* = *ax*2 + *bx* + *c*, where *a* ≠ 0, is a curve
called a parabola.

for *a* < 0

for *a* > 0

* Parabolas have a line of symmetry and
a shape as shown.
* To sketch the graph of a function, find the points where the graph intersects the axes.
* To find where the curve intersects the *y*-axis substitute *x* = 0 into the function.
* To find where the curve intersects the *x*-axis substitute *y* = 0 into the function.
* At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
* To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

**Example 1** Sketch the graph of *y* = *x*2.

|  |  |
| --- | --- |
|   | The graph of *y* = *x*2 is a parabola.When *x* = 0, *y* = 0.*a* = 1 which is greater than zero, so the graph has the shape: |

**Example 2** Sketch the graph of *y* = *x*2 − *x* − 6.

|  |  |
| --- | --- |
| When *x* = 0, *y* = 02 − 0 − 6 = −6So the graph intersects the *y*-axis at (0, −6)When *y* = 0, *x*2 − *x* − 6 = 0(*x* + 2)(*x* − 3) = 0*x* = −2 or *x* = 3So, the graph intersects the *x*-axis at (−2, 0) and (3, 0)*x*2 − *x* − 6 =  = When ,  and , so the turning point is at the point  | **1** Find where the graph intersects the *y*-axis by substituting *x* = 0.**2** Find where the graph intersects the *x*-axis by substituting *y* = 0.**3** Solve the equation by factorising.**4** Solve (*x* + 2) = 0 and (*x* − 3) = 0.**5** *a* = 1 which is greater than zero, so the graph has the shape:*(continued on next page)***6** To find the turning point, complete the square.**7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero. |

Practice

**1** Sketch the graph of *y* = −*x*2.

**2** Sketch each graph, labelling where the curve crosses the axes.

 **a** *y* = (*x* + 2)(*x* − 1)

**b** *y* = *x*(*x* − 3)

**c** *y* = (*x* + 1)(*x* + 5)

**3** Sketch each graph, labelling where the curve crosses the axes.

 **a** *y* = *x*2 − *x* − 6

**b** *y* = *x*2 − 5*x* + 4

**c** *y* = *x*2 – 4

 **d** *y* = *x*2 + 4*x*

 **e** *y* = 9 − *x*2

 **f** *y* = *x*2 + 2*x* – 3

**4** Sketch the graph of *y* = 2*x*2 + 5*x* − 3, labelling where the curve crosses the axes.

Extend

**5** Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

 **a** *y* = *x*2 − 5*x* + 6

**b** *y* = −*x*2 + 7*x* − 12

**c** *y* = −*x*2 + 4*x*

**6** Sketch the graph of *y* = *x*2 + 2*x* + 1. Label where the curve crosses the axes and write down the equation of the line of symmetry.